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Disclination Motion in Hexatic and Smectic-C Films

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We present theoretical study of a single disclination motion in a thin free standing hexatic (or smectic-C) film, driven by a large-scale inhomogeneity in the bond (or director) angle. Back-flow effects and own dynamics of the bond angle are included. We find the angle and hydrodynamic velocity distributions around the moving disclination, what allows us to relate the disclination velocity to the angle gradient far from the disclination. Different cases are examined depending on the ratio of the rotational and shear viscosity coefficients.

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Introduction

Physics of thin liquid-crystalline films has been a recurrent hot topic during the past decade because of their intriguing physical properties and wide applications in display devices, sensors, and for many other purposes. Hexatic and smectic-C liquid crystalline films, we consider, belong to two-dimensional systems with spontaneously broken continuous symmetry. Therefore an essential role in the behavior of the films is played by vortex-like excitations (disclinations). Defects are almost necessarily present in liquid crystals, and their dynamics plays a crucial role in the overall pattern organization. Early studies of defects focused on classifying the static properties of the defects and their interactions [1, 2]. More recently the focus has moved to examining the dynamics of defects (see, e.g., [3] and references herein). Note that though in most practical applications of liquid crystals, such as traditional display devices, defects destroy an optical adjustment, and therefore are not desirable, there are novel display designs (bistable, multidomain liquid crystalline structures) just exploiting defect properties.

Unfortunately, theoretical researches of dynamic characteristics of the films are in a rather primitive stage, in spite of the fact that experimental dynamic studies are likely to be more fruitful. It is largely accounted for by a complexity of dynamic phenomena in the films, and a complete and unifying description of the problem is still not available. Moreover, some papers devoted to these problem (dynamics of defects) claim contradicting results. These contradictions come mainly from the fact that different authors take into account different microscopic dissipation mechanisms, but partially the source of controversy is related to semantics, since different definitions for the forces acting on defects are used (see, e.g., discussion in [4]).

In this paper we examine theoretically the disclination dynamics in free standing hexatic and smectic-C films at scales, much larger than their thickness, where the films can be treated as $2d$ objects. Our investigation is devoted to the first (but compulsory) step of defect dynamics studies: a single point disclination in a hexatic or smectic-C film. A number of theoretical efforts [5, 6, 7, 8, 9] deal with similar problems. Our justification for adding one more paper to the topic is the fact that we did not see in the literature a full investigation of the problem taking into account hydrodynamic back-flow effects. Evidently, these effects can drastically modify dynamics of defects. The goal of this work is to study the disclination motion in free standing liquid crystalline films on the basis of hydrodynamic equations, containing some phenomenological parameters (elasticity modulus, shear and rotational viscosity coefficients).

In our approach the disclination is assumed to be driven by a large-scale inhomogeneity in the bond or director angle providing a velocity for the disclination motion (relative to the film). As a physical realization of such non-uniform angle field, one can have in mind a system of disclinations distributed with a finite density. Then the inhomogeneity near a given disclination is produced by fields of other disclinations. One can also think about a pair of disclinations of opposite topological charges, then this inhomogeneity is related to mutual orientational distortion fields created by each disclination at the point of its counterpart. In fact, the majority of experimental and numerical studies of disclination motions in liquid crystals and their models [10, 11, 12, 13, 14, 15, 16, 17, 18] is devoted just to investigation of the dynamics of two oppositely charged defects. We solve the hydrodynamic equations and find bond angle and flow velocity distributions around the moving disclination. The results enable us to relate the disclination velocity and a gradient of the bond angle far from the moving disclination.

An obvious object, where our results can be applied, is the film dynamics near the Berezinskii-Kosterlitz-Thouless phase transition. The static properties of the films near the transition have been investigated in a huge number of

papers starting from the famous papers by Berezinskii [19] and Kosterlitz and Thouless [20]. There is some literature discussing the theory of dynamic phenomena associated with vortex-like excitations in condensed matter physics: vortices in type-II superconductors (see, e.g., [21]), vortices in superfluid ^4He and ^3He (see, e.g., [22, 23]), dislocations in crystals and disclinations (and other topological defects) in liquid crystals (see [10, 11, 12, 13, 14, 24, 25, 26, 27]). However, most of the theoretical works on the topic start from phenomenological equations of motion of the defects, and our aim is to derive the equations and to check their validity.

Our paper has the following structure. Section I contains basic hydrodynamic equations for liquid crystalline films necessary for our investigation. In Section II we find the bond angle and the flow velocity around the uniformly moving disclination in different regions, that allows us to relate the disclination velocity to the angle gradient far from the disclination. Different cases, depending on the ratio of the rotational and shear viscosity coefficients, are examined in Section III. The last section IV contains summary and discussion. Appendix is devoted to details of calculations of the velocity and bond angle fields around the moving disclination. Those readers who are not very interested in mathematical derivations can skip this Appendix finding all essential physical results in the main text of the paper.

I. BASIC RELATIONS FOR HEXATIC AND SMECTIC-C FILMS

Let us formulate basic relations needed to describe disclination motion in thin liquid crystalline films. Here we investigate freely suspended films on scales, larger than the thickness of the films, where they can be treated as two-dimensional objects. Such object can be described in terms of a macroscopic approach, containing some phenomenological parameters.

There are different types of the film ordering. Experimentally, hexatic and smectic-C films are observed. In these films, as in $3d$ nematic liquid crystals, the rotational symmetry is spontaneously broken. The smectic-C films are characterized by the liquid crystalline director which is tilted with respect to the normal to the film, therefore there is a specific direction along the film. In the hexatic films molecules are locally arranged in a triangular lattice, however the lattice is not an ideal one. The positional order does not extend over distances larger than a few molecular size. Nevertheless, the bond order extends over macroscopic distances. Therefore the phase is characterized by the D_{6h} point group symmetry. The order parameter for the smectic-C films is an irreducible tensor $Q_{\alpha\beta}$ (the subscripts denoted by Greek letters run two values, since we treat the films as $2d$ objects). The hexatic order parameter is an irreducible symmetric sixth-rank tensor $Q_{\alpha\beta\gamma\delta\mu\nu}$ [28], both such tensors have two independent components. Note that the order in the smectic-C films can be readily observed, by looking for in-plane anisotropies in quantities such as the dielectric permeability tensor. Because of its intrinsic sixfold rotational symmetry, hexatic orientational order is hardly observable. However, it can be detected, e.g., as a sixfold pattern of spots in the in-plane monodomain X-ray structure factor, proportional to $Q_{\alpha\beta\gamma\delta\mu\nu}$ (see, e.g., [3] and references herein).

In accordance with the Goldstone theorem, in the both cases of smectics-C and hexatics the only "soft" degree of freedom of the order parameter is relevant on large scales, which is an angle φ (like the phase of the order parameter for the superfluid ^4He). It is convenient to express a variation of the order parameter in terms of a variation of the angle φ. For the smectic-C films the relation is

$$\delta Q_{\alpha\beta} = -\delta\varphi\epsilon_{\alpha\mu}Q_{\mu\beta} - \delta\varphi\epsilon_{\beta\mu}Q_{\alpha\mu}\,, \tag{1.1}$$

where $\epsilon_{\alpha\rho}$ is the two-dimensional antisymmetric tensor. For the hexatic films

$$\delta Q_{\alpha\beta\gamma\delta\mu\nu} = -\delta\varphi\epsilon_{\alpha\rho}Q_{\rho\beta\gamma\delta\mu\nu} + \dots\,, \tag{1.2}$$

where dots represent a sum of all possible combinations of the same structure. Thus for the both films symmetries the order parameter can be characterized by its absolute value $|Q|$ and the phase φ which are traditionally represented as a complex quantity Ψ (see, e.g., [24]). For the hexatic films the quantity is written as $\Psi = |Q|\exp(6i\varphi)$, and for the smectic-$C$ films the quantity is written as $\Psi = |Q|\exp(2i\varphi)$.

The angle φ should be included into the set of the macroscopic variables of the films. A convenient starting point of the consideration is the energy density (per unit area) $\rho v^2/2 + \varepsilon$, where ρ is the $2d$ mass density, $\boldsymbol{v}$ is the film velocity, and ε is the internal energy density. The latter is a function of the mass density ρ, the specific entropy σ, and the angle φ. In fact, ε depends on $\nabla\varphi$, since any homogeneous shift of the angle φ does not touch the energy. For the hexatic films, leading terms of the energy expansion over gradients of φ are

$$\varepsilon = \varepsilon_0(\rho,\sigma) + \frac{K}{2}(\nabla\varphi)^2, \tag{1.3}$$

where K is the only (due to hexagonal symmetry) orientational elastic module of the film. For smectic-C films one should introduce two orientational elastic modules, longitudinal and transversal with respect to the director $\boldsymbol{n}$,

determining a specific direction in the film: $Q_{\alpha\beta} \propto \delta_{\alpha\beta} - 2n_\alpha n_\beta$. However, the anisotropy in real smectic-C films is weak because of the small tilt angle of the director. Moreover, fluctuations of the director lead to a renormalization of the modules, and on large scales there occurs an isotropization of a smectic-C film [29]. Therefore in this case we can use the same expression (1.3) for the elastic energy.

The complete dynamic equations for the freely suspended liquid crystalline films, valid on scales larger than their thickness, can be found in [30]. We are going to treat a quasistationary motion of the disclination. At such motions hard degrees of freedom are not excited. By other words, we can accept incompressibility and neglect bending deformations which are suppressed by the presence of the surface tension in freely suspended films. In the same manner, for the quasistationary disclination motion the thermo-diffusive mode is not excited, what implies the isothermal condition. For freely suspended films such effects as substrate friction (relevant, say, for Langmuir films) are absent. Thus at treating the disclination motion we can consider the system of equations for the velocity $\boldsymbol{v}$ and the angle φ solely. The equations are formulated at the conditions $\rho = \text{const}$, $T = \text{const}$ (where T is temperature) and $\nabla \boldsymbol{v} = 0$.

The equation for the velocity follows from the momentum density $\boldsymbol{j} = \rho\boldsymbol{v}$ conservation law

$$\partial_t j_\alpha = -\nabla_\beta \left[T_{\alpha\beta} - \eta(\nabla_\alpha v_\beta + \nabla_\beta v_\alpha)\right] , \tag{1.4}$$

where $T_{\alpha\beta}$ is the reactive (non-dissipative) stress tensor and η is the $2d$ shear viscosity coefficient of the film. For the two-dimensional hexatics the reactive stress tensor is (see [30], chapter 6)

$$T_{\alpha\beta} = \rho v_\beta v_\alpha - \varsigma\delta_{\alpha\beta} + K\nabla_\alpha\varphi\nabla_\beta\varphi - \frac{K}{2}\epsilon_{\alpha\gamma}\nabla_\gamma\nabla_\beta\varphi - \frac{K}{2}\epsilon_{\beta\gamma}\nabla_\gamma\nabla_\alpha\varphi , \tag{1.5}$$

where $\varsigma = \varepsilon - \rho\partial\varepsilon/\partial\rho$ is the surface tension. Note that the ratio $K\rho/\eta^2$ is a dimensionless parameter which can be estimated by substituting $3d$ quantities instead of $2d$ ones (since all the $2d$ quantities can be estimated as corresponding $3d$ quantities multiplied by the film thickness, and the latter drops from the ratio). For all known liquid crystals the ratio is $10^{-3} \div 10^{-4}$ (see, e.g., [1, 2, 3, 31]), and can be, consequently, treated as a small parameter of the theory.

The second dynamic equation, the equation for the bond angle, is

$$\partial_t\varphi = \frac{1}{2}\epsilon_{\alpha\beta}\nabla_\alpha v_\beta - v_\alpha\nabla_\alpha\varphi + \frac{K}{\gamma}\nabla^2\varphi , \tag{1.6}$$

where γ is the so-called $2d$ rotational viscosity coefficient. We did not find in the literature values of the coefficient for thin liquid crystalline films. For bulk liquid crystals (see, e.g., [1, 2, 3, 31]) the $3d$ rotational viscosity coefficient is usually few times larger than the $3d$ shear viscosity coefficient. Therefore one could expect $\gamma > \eta$. However, to span a wide range of possibilities below we treat $\Gamma = \gamma/\eta$ as an arbitrary parameter.

If disclinations are present in the film, it is no longer possible to define a single-valued continuous bond-angle variable φ. However, the order parameter is a well-defined function, which goes to zero at a disclination position. The gradient of $\varphi(t, \boldsymbol{r})$ is a single-valued function of $\boldsymbol{r}$ which is analytic everywhere except for an isolated point which is the position of the disclination. The phase acquires a certain finite increment at each turn around the disclination

$$\oint \mathrm{d}r_\alpha\, \nabla_\alpha\varphi = 2\pi s , \tag{1.7}$$

where the integration contour is a closed loop around the disclination position, going anticlockwise, and s is a topological charge of the disclination. For hexatic ordering $s = (1/6)n$, and for nematic orientational order $s = (1/2)n$, where n is an integer number. One can restrict oneself to the disclinations with the unitary charge n (i.e. with $s = \pm 1/6$ for the hexatic films, $s = \pm 1/2$ for the smectic-C films) only, since disclinations with larger $|s|$ possess a larger energy than the set of unitary disclinations with the same net topological charge, and thus the defects with larger charges are unstable with respect to dissociation to unitary ones. Therefore the disclinations with the charges $|n| > 1$ do not play an essential role in the physics of the films [1, 2, 3, 31]. To bring expressions, given below, into a compact form, we keep the notation s for the topological charge.

The static bond angle is determined by the stationary condition $\delta E/\delta\varphi = 0$, where

$$E = \int \mathrm{d}^2 r \, \left(\frac{\rho}{2}v^2 + \varepsilon\right)$$

is the energy of the film. For the energy density (1.3) the condition is reduced to the Laplace equation

$$\nabla^2\varphi = 0 . \tag{1.8}$$

There is a symmetric solution φ_0 for an isolated motionless disclination which satisfies Eqs. (1.7,1.8), which has the following gradient

$$\nabla_\alpha \varphi_0 = -s\epsilon_{\alpha\beta}\frac{r_\beta - R_\beta}{(\boldsymbol{r}-\boldsymbol{R})^2}. \tag{1.9}$$

Here $\boldsymbol{R}$ is a position of the disclination. If the origin of the reference system is placed to this point then one can write $\varphi_0 = s\arctan(y/x)$, where x and y are coordinates of the observation point $\boldsymbol{r}$. If some dynamic processes in the film occur, then φ varies in time and the distribution (1.9) is disturbed. It is perturbed also due to the presence of the bond angle distortion related to boundaries or other disclinations.

Below, we have in mind a case, when a system of a large number of disclinations (with an uncompensated topological charge) is created. For the $3d$ nematics it can be done rather easily [1, 2, 3], since due to the intrinsic elastic anisotropy the energies of positively and negatively charged defects are different. We do not aware of experimental or theoretical studies of defect nucleation mechanisms in free standing smectic or hexatic films. Hopefully, a situation with a finite $2d$ density of defects can be realized for the films as well (for instance, the defects could appear even spontaneously as a mechanism to relieve frustrations in chiral smectic or hexatic films, i.e. analogously to the formation of the Abrikosov vortex lattice in superconductors [32]). Examining a motion of the disclination in this case, we investigate a vicinity of the disclination of the order of the inter-disclination distance. Then far from the disclination the bond angle φ can be written as a sum $\mathrm{const} + \boldsymbol{ur}$, where u is much larger than the inverse inter-disclination distance (since the number of disclinations is large). Near the disclination position the bond angle φ can be approximated by the expression (1.9). Our main problem is to establish a general coordinate dependence of φ and $\boldsymbol{v}$, which, particularly, enables one to relate the bond gradient $\boldsymbol{u}$ and a velocity of the disclination.

II. FLOW AND ANGLE FIELD AROUND A UNIFORMLY MOVING DISCLINATION

Here we proceed to the main subject of our consideration, which is a single disclination, which is driven by a large-scale inhomogeneity in the bond angle φ. A disclination velocity is determined by an interplay of the hydrodynamic back-flow and by the own dynamics of the angle φ. To find the velocity, one has to solve the system of equations (1.4,1.5,1.6) with the constraint (1.7), requiring a suitable asymptotic behavior. As we explained in the previous section, at large distances from the disclination, the angle φ is supposed to behave as $\mathrm{const} + \boldsymbol{ur}$. In addition, we are going to work in the reference system, where the film as a whole is at rest. That means, that the flow velocity, excited by the disclination, has to tend to zero far from the disclination.

Let us consider a situation, when the disclination moves with a constant velocity $\boldsymbol{V}$. In the case both the angle φ and the flow velocity are functions of $\boldsymbol{r} - \boldsymbol{V}t$ (where $\boldsymbol{R} = \boldsymbol{V}t$ is the disclination position). Then the equation for the velocity (1.4) is written as

$$\rho(V_\beta - v_\beta)\nabla_\beta v_\alpha + \eta\nabla^2 v_\alpha + \frac{K}{2}\epsilon_{\alpha\beta}\nabla_\beta\nabla^2\varphi - K\nabla_\alpha\varphi\nabla^2\varphi + \nabla_\alpha\left[\varsigma - \frac{K}{2}(\nabla\varphi)^2\right] = 0\,. \tag{2.1}$$

One can omit the first (inertial) term in the left-hand side of (2.1), which is small because of the smallness of the parameter $K\rho/\eta^2$. Then we obtain from Eqs. (1.4-1.5)

$$\nabla^2 v_\alpha + \frac{K}{2\eta}\epsilon_{\alpha\beta}\nabla_\beta\nabla^2\varphi - \frac{K}{\eta}\nabla_\alpha\varphi\nabla^2\varphi + \nabla_\alpha\varpi = 0\,, \tag{2.2}$$

where $\varpi = \eta^{-1}[\varsigma - (K/2)(\nabla\varphi)^2]$. The equation for the angle φ, following from Eq. (1.6), at the same conditions is

$$\nabla^2\varphi + \frac{\gamma}{K}V_\alpha\nabla_\alpha\varphi = \frac{\gamma}{K}v_\alpha\nabla_\alpha\varphi - \frac{\gamma}{2K}\epsilon_{\alpha\beta}\nabla_\alpha v_\beta\,. \tag{2.3}$$

We are looking for a solution, which is characterized by the following asymptotic behaviour at $r\to\infty$: the velocity $\boldsymbol{v}$ vanishes and $\nabla\varphi$ tends to a constant vector $\boldsymbol{u}$. It is clear from the symmetry of the problem, that the gradient $\boldsymbol{u}$ of the bond angle is directed along the Y-axis, provided the velocity is directed along the X-axis. Therefore $\varphi \to uy$ at $r\to\infty$. Our problem is to find a relation between the quantities V and u, that is between the disclination velocity and the bond angle gradient far from the disclination. There are two different regions: large distances $r \gg u^{-1}$ and the region near the disclination $r \ll u^{-1}$. At large distances corrections to the leading behavior $\varphi \approx uy$ are small and the problem can be treated in the linear over these corrections approximation. In the region near the disclination φ is close to the static value (1.9) and the flow velocity $\boldsymbol{v}$ is close to the disclination velocity $\boldsymbol{V}$ (the special case when

the ratio γ/η is extremely small will be discussed in Subsection III C). Those two regions are separately examined below. The relation between u and V can be found by matching the asymptotics at $r \sim u^{-1}$. As a result, one obtains

$$V = \frac{K}{\eta} C u \,, \tag{2.4}$$

where C is a dimensionless factor depending on the dimensionless ratio $\mathit{\Gamma} = \gamma/\eta$. This factor C is of the order of unity if $\mathit{\Gamma} \sim 1$. We are interested in the asymptotic behavior of C at small and large $\mathit{\Gamma}$.

A. Region near the disclination

Let us consider the region $r \ll u^{-1}$. Here one can write

$$\varphi = \varphi_0(\boldsymbol{r} - \boldsymbol{R}) + \varphi_1(\boldsymbol{r} - \boldsymbol{R}) \,, \tag{2.5}$$

where $\boldsymbol{R} = \boldsymbol{V} t$ is the disclination position, the angle φ_0 corresponds to an equilibrium bond angle around an unmoving disclination, and φ_1 is a small correction to φ_0. The gradients of φ_0 are determined by Eq. (1.9).

Linearizing the equations (2.2,2.3) in φ_1, one obtains

$$\eta \nabla^2 v_\alpha + \frac{K}{2} \epsilon_{\alpha\beta} \nabla_\beta \nabla^2 \varphi_1 - K \nabla_\alpha \varphi_0 \nabla^2 \varphi_1 + \nabla_\alpha \left[\varsigma - \frac{K}{2} (\nabla \varphi)^2 \right] = 0 \,, \tag{2.6}$$

$$\nabla^2 \varphi_1 - \frac{\gamma}{K} v_\alpha \nabla_\alpha \varphi_0 + \frac{\gamma}{2K} \epsilon_{\alpha\beta} \nabla_\alpha v_\beta = -\frac{\gamma}{K} V_\alpha \nabla_\alpha \varphi_0 \,. \tag{2.7}$$

Introducing a new variable $\chi = (K/\eta) \nabla^2 \varphi_1$ we rewrite Eqs. (2.6,2.7) as

$$\nabla^2 v_\alpha + \frac{1}{2} \epsilon_{\alpha\beta} \nabla_\beta \chi - \nabla_\alpha \varphi_0 \chi + \nabla_\alpha \varpi = 0 \,, \tag{2.8}$$

$$\chi - \mathit{\Gamma} v_\alpha \nabla_\alpha \varphi_0 + \frac{\mathit{\Gamma}}{2} \epsilon_{\alpha\beta} \nabla_\alpha v_\beta = -\mathit{\Gamma} V_\alpha \partial_\alpha \varphi_0 \,, \tag{2.9}$$

where as above $\mathit{\Gamma} = \gamma/\eta$ and $\varpi = \eta^{-1} \left[\varsigma - K/2 (\nabla \varphi)^2 \right]$. It follows from Eq. (2.8) and $\nabla_\alpha v_\alpha = 0$ that $\nabla^2 \varpi = \nabla_\alpha \varphi_0 \nabla_\alpha \chi$. A solution of the system (2.8-2.9) can be written as

$$v_\alpha = V_\alpha + \epsilon_{\alpha\beta} \nabla_\beta \Omega \,, \tag{2.10}$$

where V_α is an obvious (due to Galilean invariance) forced solution and the stream function Ω describes a zero mode of the system (2.8-2.9). The system is homogeneous in r, and therefore Ω is a sum of contributions which are power-like functions of r.

Taking curl of Eq. (2.8) we obtain

$$-\nabla^4 \Omega - \frac{1}{2} \nabla^2 \chi - \epsilon_{\gamma\alpha} \nabla_\alpha \varphi_0 \nabla_\gamma \chi = 0 \,. \tag{2.11}$$

Substituting χ expressed in terms of $\boldsymbol{v}$ from Eq. (2.9) into Eq. (2.11), and using explicit expressions for derivatives of φ_0, in the polar coordinates (r, ϕ) we obtain

$$\left(1 + \frac{\mathit{\Gamma}}{4} \right) \nabla^4 \Omega + s \mathit{\Gamma} \left(\frac{2}{r^2} \partial_r^2 \Omega - \frac{1}{r^2} \nabla^2 \Omega - s \frac{1}{r^2} \partial_r^2 \Omega + s \frac{1}{r^3} \partial_r \Omega \right) = 0 \,. \tag{2.12}$$

Solutions of Eq. (2.12) are superpositions of terms $\propto r^{\alpha+1} \exp(im\phi)$. Substituting this r, ϕ-dependence into Eq. (2.12), one obtains the equation for α, which has the following roots

$$\alpha = \pm \frac{1}{\sqrt{2}} \left[2 + 2m^2 - s(1-s)\tilde{\mathit{\Gamma}} \pm \sqrt{\left(2 + 2m^2 - s(1-s)\tilde{\mathit{\Gamma}} \right)^2 - 4s\tilde{\mathit{\Gamma}}(m^2 - 1 + s) - 4(m^2 - 1)^2} \right]^{1/2} \,, \tag{2.13}$$

where $\tilde{\mathit{\Gamma}} = \mathit{\Gamma}(1 + \mathit{\Gamma}/4)^{-1}$. Hence $0 < \tilde{\mathit{\Gamma}} < 4$ for any γ and η. Evidently, all the roots (2.13) are real. Let us emphasize that there is no solution $\alpha = 0$ (corresponding to a logarithmic in r behavior of the velocity) among the set (2.13).

The first angular harmonic with $|m|=1$ is of particular interest because far from the disclination $\varphi_1 = ur\sin\phi$ and $\Omega = -Vr\sin\phi$. If Γ is small, there is a pair of small exponents

$$\alpha = \pm\alpha_1\,, \qquad \alpha_1 = s\sqrt{\Gamma}/2\,, \tag{2.14}$$

for $m=\pm 1$. Otherwise, for any other relevant m (terms with $m=0$ are forbidden because of the symmetry), exponents (2.13) have no special smallness.

We established that Ω is a superposition of the terms $\propto r^{\alpha+1}\exp(im\phi)$ with α determined by Eq. (2.13). Then the velocity can be found from Eq. (2.10). To avoid a singularity in the velocity at small r, one should keep contributions with positive α only. By other words, one can say, that the velocity field contains contributions with all powers α (2.13) but the factors at the terms with negative α are formed at $r\sim a$ (where a is the disclination core radius), and therefore the corresponding contributions to the velocity are negligible at $r \gg a$ (this statement should be clarified and refined for small negative exponents $-\alpha_1$ in the limit of small Γ, see below, Subsection III C). We conclude, that the correction to $\boldsymbol{V}$ in the flow velocity $\boldsymbol{v}$, related to Ω in Eq. (2.10), is negligible at $r\sim a$. Thus we come to the non-slipping condition for the disclination motion: the disclination velocity $\boldsymbol{V}$ coincides with the flow velocity $\boldsymbol{v}$ at the disclination position.

Next, to find φ, one should solve the equation $(K/\eta)\nabla^2\varphi = \chi$, where χ is determined from Eq. (2.9). Then, besides the part determined by the velocity, zero modes of the Laplacian could enter φ_1. The most dangerous zero mode is Uy, since it produces a non-zero momentum flux (and associated to it the Magnus force)

$$\oint \mathrm{d}r_\alpha\, \epsilon_{\alpha\beta} T_{\beta\gamma} \sim KU\,,$$

to the disclination core. However, because of the condition $\alpha \neq 0$, all the contributions to the velocity correspond to zero viscous momentum flux to the origin. Consequently, it is impossible to compensate the Magnus force by other terms. The above reasoning leads us to the conclusion that the factor U (and therefore the Magnus force) must be zero. Thus, φ_1 contains only terms, proportional to $r^{\alpha+1}$, with $\alpha > 0$. This conclusion is related to the fact that for free standing liquid crystalline films any distortion of the bond angle unavoidably produces hydrodynamic back-flow motions (i.e. $\boldsymbol{v}\neq 0$). Unlike free standing films, for the liquid crystalline films on substrates (Langmuir films) hydrodynamic motions (back-flows) are strongly suppressed by the substrate, and a situation when the back-flow is irrelevant for the disclination motion could be realized.

B. Remote Region

Let us consider the region $r \gg u^{-1}$, where we can write $\varphi = uy + \tilde\varphi$ and linearize the system of equations (2.2,2.3) with respect to $\tilde\varphi$. Then we get the system of linear equations for $\boldsymbol{v}$ and $\tilde\varphi$:

$$\begin{aligned} &\nabla^2 v_\alpha + \frac{K}{2\eta}\left(\epsilon_{\alpha\beta}\nabla_\beta\nabla^2\tilde\varphi - 2u_\alpha\nabla^2\tilde\varphi\right) + \nabla_\alpha\varpi = 0\,,\\ &(\nabla^2 + 2p\partial_x)\tilde\varphi + \frac{\gamma}{2K}(\epsilon_{\alpha\beta}\nabla_\alpha v_\beta - 2uv_y) = 0\,, \end{aligned} \tag{2.15}$$

where $p = V\gamma/2K$. Taking curl of the first equation and excluding the Laplacian, we obtain

$$\epsilon_{\beta\alpha}\nabla_\beta v_\alpha = \frac{K}{2\eta}\left[\left(\nabla^2 + 2u\partial_x\right)\tilde\varphi + \Phi\right]\,, \tag{2.16}$$

where Φ is a harmonic function. In terms of the harmonic function Φ the system (2.15) is reduced to

$$\left[(1+\Gamma/4)\nabla^4 + 2p\nabla^2\partial_x - \Gamma u^2\partial_x^2\right]\tilde\varphi = (\Gamma/2)u\partial_x\Phi\,. \tag{2.17}$$

The equation (2.17) can be written as

$$(\nabla^2 + 2k_1\partial_x)(\nabla^2 - 2k_2\partial_x)\tilde\varphi = (\tilde\Gamma/2)u\partial_x\Phi\,, \tag{2.18}$$

$$k_{1,2} = \frac{1}{2(1+\Gamma/4)}\left(\sqrt{p^2 + \Gamma(1+\Gamma/4)u^2} \pm p\right)\,. \tag{2.19}$$

The quantities k_1 and k_2 have the meaning of characteristic wave vectors. We conclude from Eq. (2.18) that zero modes of the operator in the left hand side of the equation are proportional to

$$\exp(-k_1 r - k_1 x)\,, \qquad \exp(-k_2 r + k_2 x)\,,$$

that is they are exponentially small everywhere, except for narrow angular regions near the X-axis. The behavior of the zero modes inside the regions is power-like in r. Besides, there is a contribution to $\tilde{\varphi}$ related to the harmonic function Φ. It contains a part which decays as a power of r (the leading term is $\propto r^{-1}$) at large $r \gg u^{-1}$. This solution is examined in more detail in Appendix A 1.

III. DIFFERENT REGIMES GOVERNED BY $\mathit{\Gamma}$

A character of the velocity and the bond fields around the moving disclination is sensitive to the ratio of the rotational and the shear viscosity coefficients $\mathit{\Gamma} = \gamma/\eta$. In this section we examine different cases, depending on the $\mathit{\Gamma}$ value.

A. The case $\mathit{\Gamma} \gtrsim 1$

We start to treat different regimes of mobility with the most likely case $\mathit{\Gamma} \gtrsim 1$. If $\mathit{\Gamma} \sim 1$ then the factor C in Eq. (2.4) is of order unity and $u \sim p$. Then we find from Eqs. (2.19), that $k_1, k_2 \sim u$. It is a manifestation of the fact, that there is the unique characteristic scale in this case, which is u^{-1}. Then one can estimate $\tilde{\varphi}$ by matching at $r \sim u^{-1}$ of the solutions in the regions near the disclination and far from it. We conclude, that it is a function of a dimensionless parameter ur; the function is of the order of unity, when its argument ur is of the order of unity.

For large $\mathit{\Gamma}$ there remains the unique characteristic scale u^{-1} and, consequently, in this case still $C \sim 1$. To prove the statement, we treat first small distances $r \ll u^{-1}$. As we have shown in Subsection II A, the corrections φ_1 and $\delta\boldsymbol{v}$, to φ_0 and $\boldsymbol{V}$ respectively, are expanded into the series over the zero modes characterized by the exponents (2.13). Particularly, for $m = 1$ we can write $\varphi_1 \sim uy(ur)^\alpha$. In the large $\mathit{\Gamma}$ limit the exponents α (2.13) are regular since $\tilde{\mathit{\Gamma}} \to 4$. From (2.13) we have $\alpha_1 \sim 1$, and for this case

$$\chi \sim \frac{K}{\eta r^2} uy(ur)^{\alpha_1} \, .$$

Comparing equations (2.8) and (2.9), we conclude that at large $\mathit{\Gamma}$ the term with χ in the second equation can be omitted and therefore the equation is a constraint imposed on the velocity. Then the first equation gives

$$|\delta v| \sim \frac{K}{\eta r} uy(ur)^{\alpha_1} \, .$$

The disclination velocity can be found now from the relation $V \sim |\delta v|$ at the scale u^{-1}, that is $p \sim \mathit{\Gamma} u$, or $C \sim 1$. The complete analysis covers also the remote region. At the condition $p \sim \mathit{\Gamma} u$ both $k_{1,2} \sim u^{-1}$. Then using the procedure of Appendix A 1, one can prove that the solutions from two regions can be matched at $r \sim u^{-1}$, and thus there are no new characteristic scales, indeed. We also note that the rotational viscosity γ is excluded from the hydrodynamic equations at large $\mathit{\Gamma}$. Although this is not true inside the disclination core provided that one can employ hydrodynamics there (see Appendix A 4), boundary conditions for $\boldsymbol{v}$ and φ on the core reveal no dramatic behavior. Consequently, it is the shear viscosity alone that determines the disclination mobility, which means that $C \sim 1$.

Thus we can say that the limit $\mathit{\Gamma} \to \infty$ is trivial: no any additional feature appears in comparison with $\mathit{\Gamma} \sim 1$. However, it is not the case for small $\mathit{\Gamma}$, since for $\mathit{\Gamma} \ll 1$ it turns out that $u \gg p$. We study this case in the next Subsection.

B. Small $\mathit{\Gamma}$

Here, we consider the case $\mathit{\Gamma} \ll 1$. This limit is attained at anomalously large η, so that $K\rho/\eta^2$ should be still treated as the smallest dimensionless parameter. That justifies use of the same equations (2.2,2.3), as in the previous subsections.

At $r \ll u^{-1}$ the analysis of Subsection II A is correct. As we noted, contributions to $\boldsymbol{v}$ and φ_1, related to the modes with negative α, should not be taken into account there. At $\mathit{\Gamma} \ll 1$ the leading role is played by the mode with the smallest exponent ($\alpha_1 = s\sqrt{\mathit{\Gamma}}/2$), because the presence of modes with positive exponents $\alpha \sim 1$ would contradict the condition of smooth matching at $r \sim u^{-1}$. Strictly speaking, neglecting a small negative exponent $-\alpha_1$ is correct at the condition $\alpha_1 |\ln(ua)| \gg 1$, where a is the core radius of the disclination. In this subsection we consider just this

case. The opposite case, that we will term it as extremely small Γ limit, is analyzed in Subsection III C. At $r \ll u^{-1}$ we can write

$$\varphi_1 \sim uy(ur)^{\alpha_1}\,, \qquad V - v_x \sim \alpha_1 u \frac{K}{\gamma}(ur)^{\alpha_1}\,, \tag{3.1}$$

where the coefficient at $y(ur)^{\alpha_1}$ is determined from matching at $r \sim u^{-1}$ where $\nabla\varphi \sim 1/r$. Analogous matching $V - v_x \sim V$ at $r \sim u^{-1}$ gives $V \sim \alpha_1 uK/\gamma$. The relation can be rewritten as $p \sim \alpha_1 u \ll u$. Therefore we conclude $C \sim 1/\sqrt{\Gamma}$.

The relation $p \sim \sqrt{\Gamma}\,u$ leads to $k_{1,2} \sim p \ll u$, as follows from Eq. (2.19). By other words, a new scale p^{-1} (different from u^{-1}) appears in the problem. Therefore a detailed investigation of the far region $r \gg u^{-1}$ is needed, to establish an $\boldsymbol{r}$-dependence of the bond angle φ and the velocity field $\boldsymbol{v}$ there. This investigation can be based on the equations, formulated in Subsection II B, which are correct irrespective to the value of pr.

Explicit expressions, describing the velocity and the angle, are presented in Appendix A 1. They contain three dimensionless functions $\zeta_1(\nabla/u)$, $c_1(\nabla/u)$ and $c_2(\nabla/u)$. Really, at $ur \gg 1$ one can keep only zero terms of the expansions of these functions into the Taylor series. Among these three coefficients only one is independent, see Eq. (A10). Therefore the general solution could be expressed with a single free parameter, $\zeta \equiv \zeta_1(0)$. The procedure corresponds to the following construction of solutions of the equations of motion (2.15) in the region $ur \gg 1$. We have to match the solutions in the outer (far from the disclination) and in the inner (close to the disclination) regions at $ur \simeq 1$. Technically the matching is equivalent to the appropriate boundary conditions for the outer problem at $ur \simeq 1$, and these boundary conditions can be replaced formally by the local source terms in the equations, acting at $ur \simeq 1$. One can expand these sources in the standard multipolar series. Thus we come to the expansion over the gradients of δ-function. The gradients scale as u, and therefore ζ, c_1, c_2 are dimensionless functions of the dimensionless ratio ∇/u.

To establish asymptotic behavior of the angle φ and of the velocity $\boldsymbol{v}$, let us first consider the region $u^{-1} \ll r \ll p^{-1}$. Then we derive from Eqs. (A4,A5,A10)

$$v_x = \frac{K(2s-\zeta)}{\gamma u} k_1 k_2 \ln(pr)\,. \tag{3.2}$$

Here we presented only the leading logarithmic contribution of the zero harmonic in v_x. Matching the velocity derivatives, determined by Eqs. (3.1,3.2), at $r \sim u^{-1}$, we find $\zeta \sim 1$ (we imply $s \sim 1$). Using the expressions (A2,A5,A10), we obtain for the region $u^{-1} \ll r \ll p^{-1}$

$$\varphi = \varphi_0 + uy + spy\ln(pr)\,. \tag{3.3}$$

We see, that there is only a small correction to the simple expression $\varphi_0 + uy$ there, since $p \ll u$.

In the region $pr \gg 1$, the expressions for the angle φ and the velocity $\boldsymbol{v}$ are more complicated. Using the formulas (A2,A3,A4,A5), one obtains

$$\partial_x\varphi = -s\sqrt{\frac{\pi}{2}}\left[c_1\sqrt{k_1}\,\exp(-k_1 r - k_1 x) + c_2\sqrt{k_2}\,\exp(-k_2 r + k_2 x)\right]\frac{y}{r^{3/2}} - \frac{\zeta}{2}\frac{y}{r^2}\,, \tag{3.4}$$

$$\partial_y\varphi = u - 2s\sqrt{\frac{\pi}{2}}\left[c_1\sqrt{k_1}\,\exp(-k_1 r - k_1 x) - c_2\sqrt{k_2}\,\exp(-k_2 r + k_2 x)\right]\frac{1}{r^{1/2}} + \frac{\zeta}{2}\frac{x}{r^2}\,, \tag{3.5}$$

$$v_y = \frac{K}{\gamma u}\left\{2s\sqrt{\frac{\pi}{2}}\left[c_1 k_2\sqrt{k_1}\,\exp(-k_1 r - k_1 x) - c_2 k_1\sqrt{k_2}\,\exp(-k_2 r + k_2 x)\right]\frac{y}{r^{3/2}} - p\zeta\frac{y}{r^2}\right\}\,, \tag{3.6}$$

$$v_x = -\frac{K}{\gamma u}\left\{\sqrt{\frac{\pi}{2}}s\Gamma u^2\left[\frac{c_1}{\sqrt{k_1 r}}\,\exp(-k_1 r - k_1 x) + \frac{c_2}{\sqrt{k_2 r}}\,\exp(-k_2 r + k_2 x)\right] + p\zeta\frac{x}{r^2}\right\}\,, \tag{3.7}$$

where $c_1 \sim 1$ and $c_2 \sim 1$ are determined by Eq. (A10) (we omitted the argument 0 to simplify the notations). The expressions (3.4,3.5,3.6,3.7) contain two types of terms: isotropic and anisotropic ones. The anisotropic contributions are essential only in the narrow angular regions near the X-axis, where they dominate. It is worth to note the very non-trivial structure of the flow, where the isotropic flux to the origin is compensated due to the anisotropic terms.

The expressions found in this subsection generalize the famous Lamb solution for the hydrodynamic flow around a hard cylinder, (see, e.g., [33, 34, 35]) where far from the cylinder the velocity field is exponentially small everywhere except in the wake of the corps, i.e. in the very narrow angular sector ("tail"). Disclination motion in liquid crystalline films can be regarded as motion of a cylinder framed by a "soft" (i.e. deformable) orientational field φ. Due to the additional (with respect to the classical Lamb problem) degree of freedom, our solution has two tails around the moving disclination: wake, beyond the disclination, and precursor in front of it. In fact, both degrees of freedom (the flow velocity and the bond angle) are relevant.

C. Extremely small $\mathit{\Gamma}$

At the above analysis we implied the condition $\alpha_1|\ln(ua)| \gg 1$ (remind that $\alpha_1 = s\sqrt{\mathit{\Gamma}}/2$ at small $\mathit{\Gamma}$), imposing a restriction from below to $\mathit{\Gamma}$ at a given u. If $\alpha_1|\ln(ua)| \ll 1$, then the terms with both $\alpha = \pm\alpha_1$, determined by Eq. (2.14), should be taken into account near the disclination, that leads to a logarithmic behavior of the correction φ_1 to φ_0 there:

$$\varphi_1 \sim uy\ln(r/a)|\ln(au)|^{-1}\,, \tag{3.8}$$

instead of Eq. (3.1). Matching the derivatives of expressions (3.3,3.8) at $r \sim u^{-1}$ gives $p \sim u|\ln(au)|^{-1}$. By other words, $C \sim [\mathit{\Gamma}\ln(au)]^{-1}$. This case corresponds formally to the limit $\eta \to \infty$ in our equations, when one can drop the back-flow hydrodynamic velocity in the equation for the bond angle. The situation was examined in the works [6, 7, 8, 9]. We present a simple analysis of the case in Appendix A 2. Note also that, really, there is no crossover at $r \sim u^{-1}$ in the bond angle behavior in this situation.

Let us clarify the question, concerning the Magnus force in this case. In accordance with Eq. (3.8) the reactive momentum flux to the disclination core is

$$\oint \mathrm{d}r_\alpha\, \epsilon_{\alpha\beta}T_{\beta\gamma} \sim Ku\ln(r/a)|\ln(au)|^{-1}\,.$$

Thus, the flux is r-dependent, tending to zero at $r \to a$. This reactive momentum flux is compensated by the viscous momentum flux, (related to derivatives the flow velocity $\boldsymbol{v}$), which is non-zero in this case because of the logarithmic in r behaviour of the flow velocity near the disclination. The flow velocity can be found from Eqs. (2.6,3.8), it behaves as

$$v_\alpha \sim \frac{Ku}{\eta|\ln(au)|}\,\epsilon_{\alpha\beta}\nabla_\beta[y\ln^2(r/a)]\,,$$

what is a generalization of the Stokes-Lamb solution [33, 34]. However unlike the Lamb problem (a hard cylinder moving in a viscous liquid), in our case $|\boldsymbol{V} - \boldsymbol{v}(r = a)| \sim V$, i.e. we have a slipping on the core of the moving disclination. This slipping seems natural in the limit of extremely small values of $\mathit{\Gamma}$, corresponding to the limit $\eta \to \infty$, that is to a strongly suppressed hydrodynamic flow. Physically, the property means that the disclination can not be understood as a hard impenetrable object in the case. It is worth noting also that found above logarithmic behavior looks similar as a general feature of the two-dimensional hydrodynamic motion, which comes from the well known fact (see e.g., [33, 34, 35]) that for two-dimensional laminar flow even for a small Reynolds number one cannot neglect nonlinear terms, which become relevant for large enough distances. However, in our case these non-linear terms do not come from the convective hydrodynamic nonlinearity, they come from the nonlinear over φ terms in the stress tensor (1.4).

An explicit expression for φ and its asymptotics, corresponding to the considered case, are found in Appendix A 2. An expression for the flow velocity field, excited by the disclination motion, is derived in Appendix A 3.

IV. CONCLUSION

Let us sum up the results of our paper. To understand physics, underlying the freely suspended film dynamics, we studied the ground case, namely, a single disclination motion in a thin hexatic (or smectic-C) film, driven by a inhomogeneity in the bond (or director) angle. We investigated the uniform motion (i.e. one with a constant velocity). For this case we derived and solved equations of motion and found bond angle and hydrodynamic velocity distributions around the disclination. It allows us to relate the velocity of the disclination V to the bond angle gradient $u = |\nabla\varphi|$ in the region far from the disclination. It is in fact the reason, why so much efforts are needed: the full set of the equations should be solved everywhere, not locally only. We established the proportionality coefficient C (see Eq. (2.4)) in this non-local relationship which has a meaning of the effective mobility coefficient. The coefficient C depends on the dimensionless ratio $\mathit{\Gamma}$ of rotational γ and shear viscosity η coefficients.

There is little experimental knowledge as far as the values of the coefficients γ and η in hexatic and smectic-C films are concerned. It is believed generally that the corresponding values in a film (normalized by its thickness) and in a bulk material are not very different [3, 31], and in this case we are in the regime of $\mathit{\Gamma} \simeq 1$, when the coefficient C is of the order of unity. However $\mathit{\Gamma} \ll 1$ case is not excluded from the both theoretical and material science points of view. We found for small $\mathit{\Gamma} \ll 1$ case the coefficient $C \sim 1/\sqrt{\mathit{\Gamma}}$. We established a highly non-trivial behaviour of the flow velocity and of the bond angle, which is power-like in r near the disclination and extremely anisotropic far from it. And only for the case of extremely small $\mathit{\Gamma} \ll 1/\ln^2(ua)$ (where a is the disclination core radius), we found a

logarithmic behavior $C \sim [\Gamma \ln(ua)]^{-1}$. The main message of our study is that the hydrodynamic motion (that is the back-flow), unavoidably accompanying any defect motion in liquid crystals, plays a significant role in the disclination mobility. Experimental evidence (see, e.g., the recent publication [36]) shows that it is indeed the case.

Our consideration can be applied to the motion of a disclination pair with opposite signs. In this case the role by the scale u^{-1} is played by the distance R between the disclinations. Then we find in accordance with Eq. (2.4) that $\partial_t R \propto R^{-1}$ without a logarithm (provided the twist viscosity coefficient γ is not anomalously small, see Subsection III C for the quantitative criterion). This conclusion is confirmed by results of numerical simulations for $2d$ nematics [15, 16, 17, 18]. The authors of the works consider the equations of motion in terms of the tensor order parameter, consistently taking into account the coupling between the disclination motion and the hydrodynamic flow. They computed dynamics of the disclination pair annihilation and found that the distance R between the disclinations scales with time t as $t^{1/2}$, without logarithmic corrections (as it follows from our theoretical consideration for any (excepting extremely small) value of the parameter Γ. Unfortunately we did not find in the publications [16, 17, 18] the magnitudes of the the shear viscosity which was used in the simulations. Lacking sufficient data on the values of γ and η we can at present discuss only the general features of the disclination dynamics. For instance the authors [18] found numerically in one-constant approximation the asymmetry of the disclination dynamics with respect to the sign of the topological charge ($s = \pm 1/2$). In our approach the asymmetry naturally appears from nonlinear terms in the stress tensor (1.5) and from the first term in the right-hand side of Eq. (1.6) responsible for the different coupling of orientational and hydrodynamic flow patterns for positive and negative disclinations. This results in the fact that smaller positive exponents in Eq. (2.13) (corresponding to the minus in the brackets) for each m are larger if $s = 1/2$ than those for $s = -1/2$. Thus, the disclination with $s = 1/2$ exerts a stronger influence on the flow velocity, the qualitative conclusion that was arrived in [18].

Although the theory, presented in the paper, is valid for free standing hexatic or smectic-C liquid crystalline films, the general scheme can be applied to Langmuir films, that is for the liquid crystalline films on solid or liquid substrates. Since the Langmuir film is arranged on the substrate surface, then any its hydrodynamic motion is accompanied by the substrate motion. Therefore for solid substrates a situation, when the hydrodynamic back-flow is irrelevant for the disclination dynamics, could be realistic. In Subsection III C (see also Appendix A 2) we examine this limit, and reproduce the results of the works [6, 7, 8, 9], where the hydrodynamic back-flow was neglected from the very beginning. As it concerns to the Langmuir films on a liquid substrate, this case requires a special investigation, however the approach and main ideas of our paper could be useful as well.

Our results can be tested directly by comparing with experimental data for smectic-C films. The hexatic order parameter, which has sixfold local symmetry is not coupled to the light in any simple way (and therefore ideal hexatic disclinations are hardly observed in optics). However it is possible to observe the core splitting of the disclinations in tilted hexatic smectic films [26]. Indeed, due to discontinuity the tilt direction (which is locked to the bond direction) one can observe hexatic order and hexatic disclinations indirectly. The second possibility of checking our theoretical results is the classical light-scattering (where in typical experiments wave vectors are $q = 10^2 \div 10^4 cm^{-1}$, and the frequency is $\omega \lesssim 10^8 s^{-1}$). For a reasonably thick film the power spectrum of light scattering can have some additional structure that reveals the disclination properties (e.g. defects are thought to be relevant to the very low-frequency noise observed in thin films). Such kind of experimental studies are highly desirable.

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APPENDIX A

Here we present some technical details, concerning the velocity and the bond angle fields around the disclination. The case of small Γ is mostly treated, which is especially rich from the point of view of the space structure of the fields.

1. Distances far from the disclination

Here we derive some results for the far from the disclination region which are afterwards applied to the case of small Γ considered in Subsection III B.

Let us examine the harmonic function Φ in Eqs. (2.16). Since the function is analytic in the region $r > u^{-1}$, it can be expanded over derivatives of $\ln r$ there. Next, due to the symmetry of the problem, Φ should be antisymmetric in y. Therefore at least one derivative ∂_y should be present in each term of the expansion, that is

$$\Phi = u\hat{\zeta}_1 \partial_y \ln r \,, \tag{A1}$$

where $\hat{\zeta}_1 = \zeta_1(\nabla/u)$, and $\zeta_1(z)$ is a series of $\boldsymbol{z}$, converging in a circle with radius of the order of unity. The expansion coefficients in the series $\zeta_1(\nabla/u)$ are determined by matching at $r \sim u^{-1}$ with the inner problem.

Because of the symmetry, the angle $\tilde{\varphi}$ can be represented in the following form:

$$\partial_x \tilde{\varphi} = \partial_y B \,, \qquad \partial_y \tilde{\varphi} = -(H + \partial_x B) \,, \qquad \nabla^2 B + \partial_x H = 0 \,. \tag{A2}$$

The latter equation is the condition $\epsilon_{\alpha\beta}\nabla_\alpha\nabla_\beta\tilde{\varphi} = 0$. Note that $\nabla^2\tilde{\varphi} = -\partial_y H$. In the far from the disclination region we can use Eqs. (2.15,2.16). The incompressibility condition $\nabla_\alpha v_\alpha = 0$ must be taken into account as well. Then one obtains expressions for the velocity in terms of B and H:

$$\operatorname{curl} \boldsymbol{v} = \frac{K}{2\eta}\partial_y \left[-H + 2uB + u\hat{\zeta}_1 \ln(pr)\right] \,,$$

$$v_y = \frac{K}{\gamma u}\partial_y \left\{-H + 2pB + \frac{\Gamma}{4}\left[-H + 2uB + u\hat{\zeta}_1 \ln(pr)\right]\right\} \,, \tag{A3}$$

$$v_x = \frac{K}{\gamma u}\left\{\partial_x \left\{-H + 2pB + \frac{\Gamma}{4}\left[-H + 2uB + u\hat{\zeta}_1 \ln(pr)\right]\right\} - \frac{\Gamma u}{2}\left[-H + 2uB + u\hat{\zeta}_1 \ln(pr)\right]\right\} \,, \tag{A4}$$

Solutions of Eq. (2.17) imply:

$$B = s\left[\hat{c}_1 K_0(k_1 r)e^{-k_1 x} + \hat{c}_2 K_0(k_2 r)e^{k_2 x}\right] - \frac{1}{2}\hat{\zeta}_1 \ln(pr) \,,$$

$$H = 2s\left[k_1\hat{c}_1 K_0(k_1 r)e^{-k_1 x} - k_2\hat{c}_2 K_0(k_2 r)e^{k_2 x}\right] \,. \tag{A5}$$

Here the particular representation (A1) is used and an arbitrary function of y which can contribute to H is chosen to be zero because $\nabla\tilde{\varphi} \to 0$ (and hence $H \to 0$) as $r \to \infty$. Above $\hat{c}_1$ and $\hat{c}_2$ are dimensionless differential operators which can be represented as Taylor series over ∇/u, i.e. $c_1(\nabla/u)$ and $c_2(\nabla/u)$. These functions must scale with u since the functions have to be found from the matching at $r \simeq u^{-1}$.

Additionally, there are two conditions on the variables in the region $ur \gg 1$. First, the correct circulation around the origin leads to the effective δ-functional term in Eq. (A2):

$$\nabla^2 B + \partial_x H = -2\pi s\delta(\boldsymbol{r}) \,. \tag{A6}$$

The second condition is the absence of the flux to the origin:

$$\int \mathrm{d}\phi \, v_r(r,\phi) = 0 \,. \tag{A7}$$

The relations (A6,A7) lead to the conditions

$$c_1(0) + c_2(0) + \zeta_1(0)/2s = 1 \,, \tag{A8}$$

$$\left(1 + \frac{\Gamma}{4}\right)\left[k_1 c_1(0) - k_2 c_2(0)\right] - \left(p + \frac{\Gamma u}{4}\right)\left[c_1(0) + c_2(0) + \zeta_1(0)/2s\right] + \frac{\Gamma u}{8s}\zeta_1(0) = 0 \,. \tag{A9}$$

At small Γ the solution of the equations (A8,A9) is

$$\zeta_1(0) = \zeta \,, \quad c_1(0) = \frac{k_1 - \zeta k_2/2s}{k_1 + k_2} \,, \quad c_2(0) = \frac{k_2 - \zeta k_1/2s}{k_1 + k_2} \,. \tag{A10}$$

We also assumed that $\zeta \lesssim 1$, which is justified in Subsection III B.

2. Suppressed Flow

Here we demonstrate, for convenience of references, how is it possible to find the disclination velocity V provided the hydrodynamic velocity $\boldsymbol{v}$ is negligible (say, due to a substrate friction). We reproduce the results of the works [6, 7, 8, 9].

In the absence of the hydrodynamic flow the equation for the angle φ is purely diffusive:

$$\gamma\partial_t\varphi = K\nabla^2\varphi\,, \tag{A11}$$

as follows from Eq. (1.6) at $\boldsymbol{v}=0$. We assume that $\varphi \to uy$ if $r\to\infty$. The disclination motion is forced by the "external field" u. Next, we are looking for a solution $\varphi(t,x,y)=\varphi(x-Vt,y)$. Then we get from Eq. (A11)

$$2p\partial_x\varphi + \nabla^2\varphi = 0\,, \quad \text{where} \quad 2p=\gamma V/K\,. \tag{A12}$$

Below we consider a solution corresponding to a single disclination with the circulation

$$\oint \mathrm{d}\boldsymbol{r}\,\nabla\varphi = 2\pi s\,, \tag{A13}$$

where the integral is taken along a contour, surrounding the disclination in the anticlockwise direction. The quantity s in Eq. (A13) is an arbitrary parameter (which is equal to $\pm 1/6$ for hexatics and $\pm 1/2$ for nematics). For a suitable solution of Eq. (A12), corresponding to Eq. (A13), we have

$$\partial_x\varphi = \partial_y \int \frac{\mathrm{d}^2 q}{2\pi}\frac{1}{q^2-2ipq_x}\exp(i\boldsymbol{q}\boldsymbol{r}) = s\exp(-px)\partial_y K_0(pr)\,. \tag{A14}$$

This derivative tends to zero at $r\to\infty$ as it should be.

The expression (A14) does not determine φ itself unambiguously, since $\partial_x(uy)=0$ and therefore one can add an arbitrary function uy to a solution and it gives a new solution then. Note that uy is a zero mode of the equation (A12). Thus the solution can be written as

$$\varphi = \phi + uy\,, \qquad \phi(x',y) = -s\int_{x'}^{\infty}\mathrm{d}x\,\exp(-px)\partial_y K_0(pr)\,, \tag{A15}$$

where ϕ tends to zero if $r\to\infty$. To relate u in Eq. (A15) and p, one must know boundary conditions at $r\to 0$, really, at $r\sim a$, where a is the core radius. At small r φ can be written as a series $\varphi=\varphi_0+\varphi_1+\dots$, where φ_0 corresponds to an motionless disclination, φ_1 is the first correction to φ_0, related to its motion. Matching with the inner problem gives

$$\nabla\varphi_1(a)\sim p\,, \tag{A16}$$

since the solution for the order parameter inside the core is an analytical function of r/a, and the expansion over p is a regular expansion over pa.

Expanding Eq. (A14) over p we get at $pr\ll 1$

$$\frac{1}{s}\partial_x\varphi \approx -\frac{y}{r^2}+\frac{pxy}{r^2}\,.$$

Therefore in accordance with Eq. (A15) one gets with the logarithmic accuracy

$$\varphi_1 = spy\ln(pr) + uy\,. \tag{A17}$$

Using now the boundary condition (A16) one gets with the same logarithmic accuracy

$$u = sp\ln\left(\frac{1}{pa}\right)\,. \tag{A18}$$

It can be rewritten as

$$V = \frac{2Ku}{s\gamma\ln(1/pa)}\,. \tag{A19}$$

The same answer (A19) can be found from the energy dissipation balance. First of all, we can find the energy E corresponding to the solution (A15):

$$E = \int \mathrm{d}^2 r \, \frac{K}{2} (\nabla \varphi)^2 = K \int \mathrm{d}^2 r \, \left[\frac{1}{2} u^2 + \frac{1}{2} (\nabla \phi)^2 + u \partial_y \phi \right] . \tag{A20}$$

Here the first term is the energy of the external field and the second term is the energy of the disclination itself. Obviously, both the terms are independent of time, and only the last cross term depends on the time. For $|x - Vt| \gg p^{-1}$

$$\int_{-\infty}^{\infty} \mathrm{d}y \, \partial_y \phi = \begin{cases} 0 & \text{if } \ x > Vt \, , \\ -2\pi s & \text{if } \ x < Vt \, . \end{cases}$$

Therefore we obtain from Eq. (A20)

$$\partial_t E = -2 s \pi K u V \, , \tag{A21}$$

On the other hand, we can use the equation (A11) to obtain

$$\partial_t E = -\frac{K^2}{\gamma} \int \mathrm{d}^2 r \, (\nabla^2 \varphi)^2 \, . \tag{A22}$$

Substituting here $\nabla^2 \varphi$ by $2p \partial_x \varphi$ in accordance with Eq. (A12) we get

$$\partial_t E = -\gamma V^2 \int \mathrm{d}^2 r \, (\partial_x \varphi)^2 \, .$$

The main logarithmic contribution to the integral comes from the region $a < r < p^{-1}$ where $\partial_x \varphi \approx -sy/r^2$. Thus we obtain

$$\partial_t E = -\pi s^2 \gamma V^2 \ln \left(\frac{1}{pa} \right) \, . \tag{A23}$$

Comparing the expression with Eq. (A21) we find the same answer (A19).

3. Extremely small Γ

Here we consider the flow velocity, excited by the moving disclination in the case of the extremely small Γ. The velocity is zero in the zero in Γ approximation (the case is considered in Appendix A 2), therefore we examine the next, first, approximation in Γ. We use the same formalism and the same designations as in Appendix A 1.

In accordance with Appendix A 1 solutions of the complete set of nonlinear stationary equations can be represented in the following form:

$$\partial_x \tilde{\varphi} = \partial_y B \, , \qquad \partial_y \tilde{\varphi} = -(H + \partial_x B) \, , \tag{A24}$$

$$\mathrm{curl} \, \boldsymbol{v} = \frac{K}{2\eta} \left[-\partial_y H + 2u \partial_y B + 2us \partial_y \ln r + \Phi' \right] \, , \tag{A25}$$

$$v_x = -\frac{K}{2\eta} \partial_y \nabla^{-2} \left[-\partial_y H + 2u \partial_y B + 2us \partial_y \ln r + \Phi' \right] \, , \tag{A26}$$

$$v_y = \frac{K}{2\eta} \partial_x \nabla^{-2} \left[-\partial_y H + 2u \partial_y B + 2us \partial_y \ln r + \Phi' \right] \, , \tag{A27}$$

where B, H and Φ' are to be found from the equations

$$\begin{aligned} &- \partial_y H + 2p \partial_y B + \frac{\Gamma}{4} \nabla^{-2} (\nabla^2 - 2u \partial_x) \left[-\partial_y H + 2u \partial_y B + 2us \partial_y \ln r + \Phi' \right] = \\ &- \frac{\Gamma}{2} \Big(\partial_y \nabla^{-2} \left[-\partial_y H + 2u \partial_y B + 2us \partial_y \ln r + \Phi' \right] \partial_y B + \partial_x \nabla^{-2} \left[-\partial_y H + 2u \partial_y B + 2us \partial_y \ln r + \Phi' \right] (\partial_x B + H) \Big) \, , \end{aligned} \tag{A28}$$

$$\Phi' = 2\nabla^{-2}[(\partial_x B + H)\partial_x\partial_y H + \partial_y B\partial_y^2 H]\,, \tag{A29}$$

$$\nabla^2 B + \partial_x H = -2\pi s\delta(\boldsymbol{r})\,. \tag{A30}$$

If Γ is extremely small, $s^2\Gamma\ln^2(ua) \ll 1$, the solution of Eqs. (A28-A30) can be continued to the vicinity of the core. In the main approximation the solution for φ coincides with the solution for the angle $\tilde{\varphi}_L$ in the absence of the liquid motion. This case, examined in [6, 7, 8, 9], is described in Appendix A 2. The functions B_L and H_L, corresponding to $\tilde{\varphi}_L$, are given by

$$2pB_L = H_L = 2spK_0(pr)\exp(-px)\,. \tag{A31}$$

This solution results in

$$\Phi' = 2s^2p\frac{y}{r^2}\ln\left(\frac{\min\{r, p^{-1}\}}{a}\right)\,. \tag{A32}$$

Then, after neglecting the nonlinear right-hand side of Eq. (A28), we can find

$$H(\boldsymbol{r}) = \frac{4\pi s}{1+\Gamma/4}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\exp(i\boldsymbol{qr})\frac{pq^2 - (s\Gamma p/4)(q^2+2iuq_x)\ln\left(\min\{(qa)^{-1},\ (pa)^{-1}\}\right)}{(q^2-2ik_1q_x)(q^2+2ik_2q_x)}\,. \tag{A33}$$

$B(\boldsymbol{r})$ can be found in the similar way. Using B and H, from Eqs. (A26,A27) we calculate such flow velocity $\boldsymbol{v}(\boldsymbol{r})$ that vanishes at infinity.

For $r \gg p^{-1}$ this solution coincides with expressions (A5,A8,A9) with

$$\zeta_1(0) = 2s + \frac{2s^2p}{u}\ln\left(\frac{1}{pa}\right)\,.$$

For $pr \ll 1$ the expression (A33) is reduced to (A31) and this region produces the main contribution to Φ' (A32). The following expressions are obtained in the inner region ($pr \ll 1$) from the solution (A24-A33):

$$\varphi_1 = \left(u - sp\ln\frac{1}{pa}\right)y + spy\ln\frac{r}{a}\,, \tag{A34}$$

$$\mathrm{curl}\ \boldsymbol{v} = \frac{Ks^2p}{\eta}\ln\frac{r}{a}\ \frac{y}{r^2}\,. \tag{A35}$$

A relation between p and u is fixed by the condition (A16), leading to $u = sp\ln[1/(pa)]$, which is equivalent to Eq. (A19). The flow velocity at $pr \ll 1$ and $\ln(r/a) \gg 1$ is

$$v_\alpha = -\frac{s^2\Gamma}{8}V\epsilon_{\alpha\beta}\nabla_\beta\left[y\ln^2(r/a)\right]\,, \tag{A36}$$

which corresponds to the stream function

$$\Omega = -Vy - \frac{Ks^2p}{4\eta}y\ln^2\left(\frac{r}{a}\right)\,. \tag{A37}$$

The expansion over Γ near the disclination is regular and can be derived from Eqs. (2.8,2.9) with the condition $\nabla\varphi_1(a) \sim p$: $\tilde{\varphi}_L + uy$ is the zero term of the series for φ, and expression (A37) represents the zero and the first terms for Ω.

Note that in accordance with Eq. (A36) in the limit $\Gamma \to 0$ the flow velocity tends to zero near the disclination core, $v(a)/V = O(\Gamma)$, despite the fact that the disclination itself moves with the finite velocity V, thus there is a slipping on the disclination core in this limit.

4. Solution with the complete order parameter

Here we consider the dynamic equations for the coupled velocity field $\boldsymbol{v}$ and the complete order parameter $\Psi = Q\exp(i\varphi/|s|)$ of the hexatic or smectic-C films. These equations are needed to examine the velocity field in a vicinity of the disclination core. Below, we work in the framework of the mean field theory.

Formally the equations can be derived, using the Poisson brackets method [30, 37]. The energy, associated with the order parameter, in the mean field approximation is

$$\mathcal{H}_\Psi = \frac{Ks^2}{2}\int \mathrm{d}^2 r \left(|\nabla\Psi|^2 + \frac{1}{2a_s^2}\left(1-|\Psi|^2\right)^2 \right) ,$$

its density passes to the K-contribution in Eq. (1.3) on large scales $r \gg a_s$. The only non-trivial Poisson bracket, which has to be added to the standard expressions (see [30, 37]), is [28]

$$\{j_\alpha(r_1), \Psi(r_2)\} = -\nabla_\alpha \Psi \delta(r_1 - r_2) + \frac{i}{2|s|}\Psi(r_2)\epsilon_{\alpha\beta}\nabla_\beta \delta(r_1 - r_2) .$$

The expression for the energy and the Poisson bracket is correct for hexatics, and turns to the above expressions at large scales for the smectics-C (due to fluctuations). Then the dynamic equations read:

$$\begin{aligned} \rho\partial_t v_\alpha + \rho v_\beta \nabla_\beta v_\alpha &= \eta\nabla^2 v_\alpha - \frac{s^2 K}{2}\left\{ \nabla_\alpha \Psi^* \left(\nabla^2\Psi + \frac{1}{a_s^2}\Psi\left(1-|\Psi|^2\right)\right) + \nabla_\alpha\Psi\left(\nabla^2\Psi^* + \frac{1}{a_s^2}\Psi^*\left(1-|\Psi|^2\right)\right)\right\} \\ &\quad - \frac{i|s|K}{4}\epsilon_{\alpha\beta}\nabla_\beta\left\{\Psi^*\nabla^2\Psi - \Psi\nabla^2\Psi^*\right\} + \nabla_\alpha\tilde{\varsigma} , \\ \partial_t\Psi + v_\alpha\nabla_\alpha\Psi &= \frac{i}{2|s|}\Psi\epsilon_{\alpha\beta}\nabla_\alpha v_\beta + \frac{Ks^2}{2\gamma_s}\left(\nabla^2\Psi + \frac{1}{a_s^2}\Psi\left(1-|\Psi|^2\right)\right) , \end{aligned} \tag{A38}$$

the relation $\gamma_s = s^2\gamma/2$ ensures the reduction to Eq. (1.6) in the limit $|\Psi| = 1$, and the kinetic coefficients are believed to be independent of Q (otherwise one can assume, for example, the dependence $\gamma_s = s^2\gamma|\Psi|^2/2$). The slow dynamics of a $2d$ liquid crystalline system with disclinations can be described by Eqs. (A38) with the additional condition of incompressibility $\nabla\boldsymbol{v} = 0$ which enables to exclude the passive variable $\tilde{\varsigma}$.

If the distance from the disclination point to another disclination is much larger than a_s, i.e. the perturbation to the static solution $\Psi_0 = Q_0\exp(i\varphi_0/|s|)$ for the single defect is small, we can linearize Eqs. (A38) with respect to the perturbation expressed in terms of corrections Q_1 and φ_1 to Q_0 and φ_0 respectively:

$$\begin{aligned} &\eta\nabla^2 v_\alpha - 2\gamma_s\left\{\nabla_\alpha Q_0(v_\beta - V_\beta)\nabla_\beta Q_0 + \frac{1}{s^2}Q_0^2\nabla_\alpha\varphi_0\left((v_\beta - V_\beta)\nabla_\beta\varphi_0 - \frac{1}{2}\epsilon_{\beta\gamma}\nabla_\beta v_\gamma\right)\right\} \\ &+ 2\gamma_s\frac{1}{2s^2}\epsilon_{\alpha\beta}\nabla_\beta\left[Q_0^2\left((v_\mu - V_\mu)\nabla_\mu\varphi_0 - \frac{1}{2}\epsilon_{\mu\nu}\nabla_\mu v_\nu\right)\right] + \nabla_\alpha\tilde{\varsigma} = 0 , \end{aligned} \tag{A39}$$

$$\frac{Ks^2}{2\gamma_s}\left(\nabla^2 Q_1 - \frac{(\nabla\varphi_0)^2}{s^2}Q_1 - \frac{2\nabla_\alpha\varphi_1\nabla_\alpha\varphi_0}{s^2}Q_0 + \frac{1}{a_s^2}\left(1-3Q_0^2\right)Q_1\right) = (v_\beta - V_\beta)\nabla_\beta Q_0 , \tag{A40}$$

$$\frac{Ks^2}{2\gamma_s}\left(\nabla^2\varphi_1 + 2Q_0^{-1}(\nabla_\alpha Q_1\nabla_\alpha\varphi_0 + \nabla_\alpha Q_0\nabla_\alpha\varphi_1)\right) = -\frac{1}{2}\epsilon_{\alpha\beta}\nabla_\alpha v_\beta + (v_\beta - V_\beta)\nabla_\beta\varphi_0 . \tag{A41}$$

In terms of dimensionless quantities $L = \eta\Omega/K$, $R = r/a_s$ and $\mathit{\Gamma} = 2\gamma_s/(s^2\eta)$, Eq. (A39) takes the following form (as previously, we consider a disclination with the unitary topological charge $|s|$ or $-|s|$):

$$\nabla_R^4 L + \frac{\mathit{\Gamma}}{4}\left\{-4s^2\frac{(\partial_R Q_0)^2}{R^2}\partial_\phi^2 L + \left(\nabla_R^2 + \frac{2s}{R}\partial_R\right)\left(Q_0^2\left(\nabla_R^2 - \frac{2s}{R}\partial_R\right)L\right)\right\} = 0 , \tag{A42}$$

Q_0 is found from

$$\left(\partial_R^2 + \frac{1}{R}\partial_R - \frac{1}{R^2}\right)Q_0 + Q_0(1-Q_0^2) = 0 , \qquad Q_0(0) = 0 , \qquad Q_0(\infty) = 1 .$$

If $\mathit{\Gamma} \gg 1$, as it follows from Eq. (A42), a new scale $R \sim 1/\sqrt{\mathit{\Gamma}} \ll 1$ appears inside the core, the first term in Eq. (A42) can be neglected on larger scales and there is no crossover at $R \sim 1$.

If $Q_0 \equiv 1$, Eq. (A42) is reduced to Eq. (2.12). If $R \ll 1$, $Q_0 = AR$ ($A \approx 0.58$), and Eq. (A42) can be rewritten as

$$\nabla_R^2\left\{\nabla_R^2 L + \frac{A\mathit{\Gamma}}{4}(R^2\nabla_R^2 - 4s^2)L\right\} = 0 .$$

The solution of the equation is a composition of terms $\lambda(R)\sin(m\phi)$ with different m. The condition $\lambda(R)=0$ keeps two constants in the general solution of the ordinary differential equation for $\lambda(R)$, two regular near $R=0$ partial solutions are

$$R^{|m|} \quad \text{and} \quad R^{|m|}\,{}_2F_1\left(\frac{|m|-\sqrt{m^2+4s^2}}{2},\, \frac{|m|+\sqrt{m^2+4s^2}}{2},\, 1+|m|,\, -\frac{A^2\Gamma R^2}{4}\right),$$

where ${}_2F_1$ is the hyper-geometric function (${}_2F_1(a,b,c,z)=1+abz/c+\ldots$). Two constants (e.g. derivatives $\lambda^{(|m|)}(0)$ and $\lambda^{(|m|+2)}(0)$) are chosen to ensure the slowest possible grow at $R\gg 1$, i.e to exclude the largest exponent among α in Eq. (2.13).

If $\Gamma\gg 1$, it is possible to derive a better approximation in the core. We can expand $Q_0(R)$ in a series, look for the series solution $\lambda(R)$ and extract the terms of the highest order in Γ. For example, for $m=1$ the series for $\lambda(R)$ begins with $l_1R+l_3R^3$, which fix two constants in the partial solution:

$$\begin{aligned}\lambda(R) =& l_1 R\left[1+\frac{1}{A^2\Gamma s(2-s^2)}\left(-1-\frac{s^2A^2\Gamma R^2}{8}+{}_2F_1\left(\frac{1-\sqrt{1+4s^2}}{2},\,\frac{1+\sqrt{1+4s^2}}{2},\,2,\,-\frac{A^2\Gamma R^2}{4}\right)\right)\right]\\ &+l_3\frac{8}{A^2\Gamma s^2}R\left[-1+{}_2F_1\left(\frac{1-\sqrt{1+4s^2}}{2},\,\frac{1+\sqrt{1+4s^2}}{2},\,2,\,-\frac{A^2\Gamma R^2}{4}\right)\right].\end{aligned}$$

The solutions of Eqs. (A40,A41) have the following form:

$$Q_1=\vartheta(R)\partial_\phi\sin(m\phi)\,, \qquad \varphi_1=\sigma(R)\sin(m\phi)\,,$$

and should be found from the equations

$$\begin{aligned}&\vartheta''+\frac{1}{R}\vartheta'-\frac{1+m^2}{R^2}\vartheta-\frac{2Q_0}{sR^2}\sigma+(1-3Q_0^2)\vartheta=\Gamma\frac{1}{R}\partial_R Q_0\lambda\,,\\ &\sigma''+\frac{1}{R}\sigma'-\frac{m^2}{R^2}\sigma+\frac{2}{Q_0}\left(-\frac{sm^2}{R^2}\vartheta+\partial_R Q_0\sigma'\right)=\frac{\Gamma}{2}\left(\lambda''+\frac{1-2s}{R}\lambda'-\frac{m^2}{R^2}\lambda\right),\end{aligned}$$

which generalize the expressions of Ref. [7].

The dynamic equations with the complex order parameter demonstrate that at any Γ nothing special happens on the core in the boundary conditions for Eqs. (1.4-1.6). The peculiarity of extremely small Γ which leads to non-slipping condition is in the slow grow of $\nabla\Omega$ far from the disclination.

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[1] P. G. de Gennes and J. Prost, The Physics of Liquid Crystals, Clarendon Press, Oxford (1995).
[2] S. Chandrasekhar, Liquid Crystals, Cambridge University Press, Cambridge (1992).
[3] P. Oswald and P. Pieranski, Les cristaux liquides, v. 1, Gordon and Breach, Paris (2000).
[4] E. B. Sonin, Phys. Rev. B **55**, 485 (1997).
[5] H. Pleiner, Phys. Rev. A **37**, 3986 (1988).
[6] E. Bodenschatz, W. Pesch, and L. Kramer, Physica D **32**, 135 (1988).
[7] L. M. Pismen and J. D. Rodriguez, Phys. Rev. A **42**, 2471 (1990).
[8] G. Ryskin and M. Kremenetsky, Phys. Rev. Lett. **67**, 1574 (1991).
[9] C. Denniston, Phys. Rev. B **54**, 6272 (1996).
[10] R. Pindak, C. Y. Young, R. B. Meyer, and N. A. Clark, Phys. Rev. Lett. **45**, 1193 (1980).
[11] R. Loft and T. A. DeGrand, Phys. Rev. B **35**, 8528 (1987).
[12] P. E. Cladis, W. van Saarloos, P. L. Finn, and A. R. Kortan, Phys. Rev. Lett. **58**, 222 (1987).
[13] A. Pargellis, N. Turok, and B. Yurke, Phys. Rev. Lett. **67**, 1570 (1991).
[14] B. Yurke, A. N. Pargellis, T. Kovacs, and D. A. Huse, Phys. Rev. E **47**, 1525 (1993).
[15] J.-i. Fukuda, Eur. Phys. J. B **1**, 173 (1998).
[16] C. Denniston, E. Orlandini, and J. M. Yeomans, Europhys. Lett. **52**, 481 (2000).
[17] C. Denniston, E. Orlandini, and J. M. Yeomans, Phys. Rev. E **64**, 021701 (2001).
[18] G. Tóth, C. Denniston, and J. M. Yeomans, Phys. Rev. Lett. **88**, 105504 (2002).
[19] V. L. Berezinskii, ZhETF **59**, 907 (1970) (Sov. Phys. JETP **32**, 493 (1971)).
[20] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[21] G. Blatter, M. V. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).

[22] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. London, ser. A **238**, 204 (1956).
[23] E. B. Sonin, Rev. Mod. Phys. **59**, 87 (1987).
[24] A. Zippelius, B. I. Halperin, and D. R. Nelson, Phys. Rev. B **22**, 2514 (1980).
[25] D. H. van Winkle and N. A. Clark, Bull. Am. Phys. Soc. **30**, 379 (1985).
[26] S. B. Dierker, R. Pindak, and R. B. Meyer, Phys. Rev. Lett. **56**, 1819 (1986).
[27] C. D. Muzny and N. A. Clark, Phys. Rev. Lett. **68**, 804 (1992).
[28] E. V. Gurovich, E. I. Kats, and V. V. Lebedev, ZhETF **100**, 855 (1991) [Sov. Phys. JETP **73**, 473 (1991)].
[29] D. R. Nelson and R. A. Pelcovits, Phys. Rev. B **16**, 2191 (1977).
[30] E. I. Kats and V. V. Lebedev, Fluctuational Effects in the Dynamics of Liquid Crystals, Springer-Verlag, New York (1993).
[31] L. M. Blinov and V. G. Chigrinov, Electrooptic Effects in Liquid Crystal Materials, Springer-Verlag, New York (1996).
[32] R. D. Kamien and J. V. Selinger, J. Phys.: Condens. Mattter **13**, R1 (2001).
[33] G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press (1970).
[34] H. Lamb, Hydrodynamics, Cambridge University Press (1975).
[35] L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon Press, Oxford (1987).
[36] E. J. Acosta, M. J. Towler, and H. G. Walton, Liquid Crystals **27**, 977 (2000).
[37] I. E. Dzyaloshinskii and G. E. Volovik, Ann. Phys. **125**, 67 (1980).